Power Amplifier Modeling Using Memory Polynomial Based on Moving Averages with Sparse Nonlinearities and Delays

Maria Eduarda Rizo
Group of Integrated Circuits and Systems (GICS)
Federal University of Paraná
Curitiba, Brazil
rizo@ufpr.br

Eduardo Gonçalves de Lima
Group of Integrated Circuits and Systems (GICS)
Federal University of Paraná
Curitiba, Brazil
eduardo.lima@ufpr.br

***Abstract*— This study addresses the behavioral modeling of power amplifiers (PAs). The objective of this work is to analyze the potential benefits in terms of improving the trade-off between accuracy and complexity by using a memory polynomial (MP) with sparse nonlinearities and delays and also with the memories values based on the moving averages of the said signal. The complete and sparse MP models were implemented in MATLAB using floating point double-precision arithmetic, as well as the moving average, and with that we were able to estimate the direct and inverse transfer characteristics of a class AB PA. The number of coefficients was fixed at 12, and it was explored variations in power and memory values. The results were analyzed using the normalized mean square error (NMSE). The introduction of moving averages allowed us to reduce NMSE by up to 0.3514 dB compared to not using it. The lowest error, -27.5138 dB, was achieved in the direct modeling estimation.**

Keywords*— mathematical modeling, sparse delays, moving averages, memory polynomial, power amplifier.

I. Introduction

Power amplifiers (PAs) are essential components in modern wireless communication systems, serving to boost the input signal's strength, ensuring reliable communication, high quality, and extended signal coverage. Mathematical modeling, a common approach, employs mathematical structures and equations to represent real-world situations, aiding in scenario prediction [1]. Recognizing the significance of PAs, this study aims to simulate PA behavior by utilizing input and output measurements alongside a mathematical model. Two modeling approaches, direct and inverse, were applied, each illustrated in Figure 1, showcasing their distinctions.

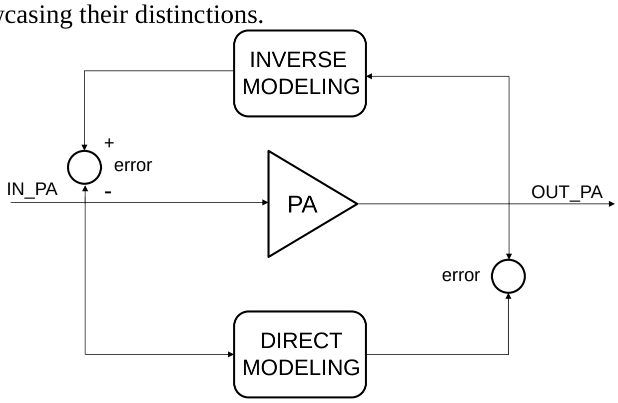

Fig. 1. The difference between direct modeling and inverse modeling.

From Figure 1, we can perceive that in direct modeling, the input of our model is the input of the PA itself, while the output is the output of our PA, with an error between the measured and the estimated output of the PA that was calculated at the end of the work. In inverse modeling, on the other hand, the input of the model is the output of the PA and the output of the model is the input of the PA, with an error between the measured and estimated output of the PA that was also calculated throughout the work.

When we have nonlinearities and the effects of memory are small, an alternative is the use of Memory Polynomials (MP) [2]. Since MP models are linear in coefficients, they are simpler to understand and interpret, as well as much easier to implement. This is precisely why they were used in this study. However, when polynomial orders are high and memory effects are long, the modeling accuracy decreases. Therefore, we fixed the number of coefficients at 12, which is P*(M+1), where P and M are the polynomial order truncation and the memory length, respectively. This means we have three scenarios: when P = 2 and M = 5, when P =3 and M = 3 and when P = 4 and M = 2.

Previously, the use of sparse delays have been suggested [3] and also the use of a MP with sparse nonlinearities, with the simultaneous application of sparse nonlinearities and delays [4] to enhance the balance between accuracy and intricacy. However, the objective of this work is to analyze the potential benefits, in terms of enhancing the balance between accuracy and complexity of using a MP with sparse nonlinearities, particularly with the simultaneous application of sparse nonlinearities and delays with the memories values based on moving averages of the sampled signal. Moving averages is a mathematical technique that computes the average of a group of values within a moving window [5]. Utilizing moving averages based on past instances enables us to capture the temporal traits and memory impacts of the system efficiently.

II. Review of the Literature

Originally [2], a MP with all powers of amplitude, that is linear in its coefficients, was proposed. This MP was used in this work, and it has the following equation:

$$\tilde{y}(n) = \sum_{p=1}^{P}\sum_{m=0}^{M} \tilde{b}_{p,m}|\tilde{x}(n-m)|^{p-1}\tilde{x}(n-m) \qquad (1)$$

where P and M are the polynomial order truncation and the memory length, respectively, $\tilde{x}(n)$ and $\tilde{y}(n)$ are the complex-envelope at the PA input and output, respectively and $\tilde{b}_{2p-1,m}$ are complex-valued coefficients.

Table I aims to exemplify what would be the sparsity in memory and in power, where we have the output $\tilde{y}(n)$ of equation (1) for when P = 2 and M = 1, for the complete MP, P = 2 and M = 2 for the MP sparse in memory and P = 3 and M = 1 for the MP sparse in power.

Table I. The difference between a complete MP, a sparse MP in memory and a sparse MP in power.

COMPLETE MP	$\tilde{y}\,(n) = \tilde{b}_{1,0}\lvert\tilde{x}(n)\rvert^{0}\tilde{x}(n) + \tilde{b}_{1,1}\lvert\tilde{x}(n-1)\rvert^{0}\tilde{x}(n-1) + \tilde{b}_{2,0}\lvert\tilde{x}(n)\rvert^{1}\tilde{x}(n) + \tilde{b}_{2,1}\lvert\tilde{x}(n-1)\rvert^{1}\tilde{x}(n-1)$
MP SPARSE IN MEMORY	$\tilde{y}\,(n) = \tilde{b}_{1,0}\lvert\tilde{x}(n)\rvert^{0}\tilde{x}(n) + \tilde{b}_{1,2}\lvert\tilde{x}(n-2)\rvert^{0}\tilde{x}(n-2) + \tilde{b}_{2,0}\lvert\tilde{x}(n)\rvert^{1}\tilde{x}(n) + \tilde{b}_{2,2}\lvert\tilde{x}(n-2)\rvert^{1}\tilde{x}(n-2)$
MP SPARSE IN POWER	$\tilde{y}\,(n) = \tilde{b}_{1,0}\lvert\tilde{x}(n)\rvert^{0}\tilde{x}(n) + \tilde{b}_{1,1}\lvert\tilde{x}(n-1)\rvert^{0}\tilde{x}(n-1) + \tilde{b}_{3,0}\lvert\tilde{x}(n)\rvert^{2}\tilde{x}(n) + \tilde{b}_{3,1}\lvert\tilde{x}(n-1)\rvert^{2}\tilde{x}(n-1)$

After analyzing the table, we can notice that when the MP is sparse in memory, there are missing memory elements, which in this work are represented by the elements varying from $\tilde{x}(n-m)$, when compared to the complete MP. Whereas when the MP is sparse in power, it is evident that, compared to the complete MP, power elements are missing, which in this work are represented by elements varying from $\lvert\tilde{x}(n-m)\rvert^{p-1}$. The sparsity was applied either to power or simultaneously to both power and delay.

The execution of the moving average proceeded in accordance with the guidelines depicted in Figure 2, and this implementation occurred for the MP with all powers, as well as for both direct and inverse modeling.

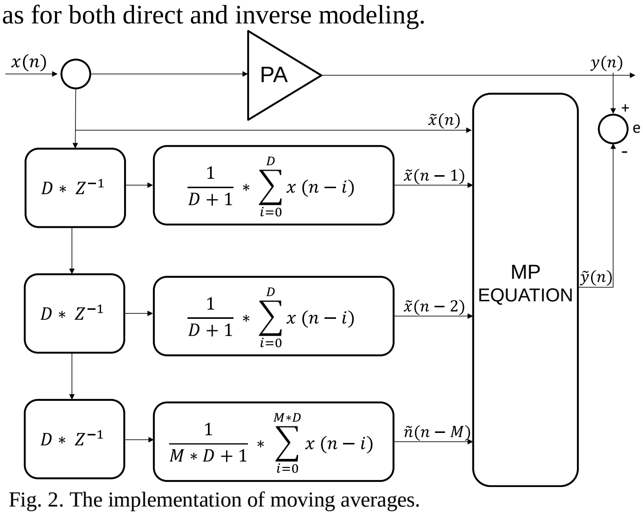

Fig. 2. The implementation of moving averages.

It was used a uniform window size for all the moving averages, and we varied the value of D from 1 to 6 throughout the simulations. This enables us to capture the temporal traits and memory impacts of the system efficiently and find out which value of D results in the best model possible. The results were shown for the best value of D corresponding to the respective modeling.

The use of moving averages enables us to accurately compare short-term and long-term memory effects and the temporal characteristics of the power amplifier. The proposed model, depicted in Fig. 2 as a simplified block diagram, incorporates the equation of the MP, in this case it would be the equation (1), running in parallel with the PA to be modeled, where $D * Z^{-1}$ represents a unit delay block applied D times. In this method, M denotes the number of moving averages used, M · D indicates the memory length of the model and e is the error. Since the parameters M and P of the series remain constant, the number of coefficients in the series does not change.

The maximum polynomial order is 5, it starts at 1 and goes until 5, and the maximum memory duration is also 5, except it starts at 0 and then goes until 5, which means we have 5 values for P and 6 values for M. These selections were made arbitrarily with the aim of attaining models of equivalent complexity, as measured by the number of coefficients. This ensures a fair comparison across the various approaches.

Regarding the resulting models, we would have what is presented in Table II, for values that are fixed or varying, where S means the number of scenarios we would have.

Table II. Number of scenarios when the values of M and P are fixed or varying.

	VALUES FIXED	**VALUES VARYING**
POWER	S = 1	$S_{P}^{5} = \frac{5!}{P! * (5-P)!}$
MEMORY	S = 1	$S = \frac{6!}{M! * (6-M)!}$

It is fair to note that when we only have one scenario of power and memory, we would have the resulting scenarios (1, 2, 3) and (0, 1, 2), assuming three different powers or delays, respectively for P and M. And for values that are varying we would have, for example for when P = 3, 10 scenarios and those would be: (1, 2, 3); (1, 2, 4); (1, 2, 5); (1, 3, 4); (1, 3, 5); (1, 4, 5); (2, 3, 4); (2, 3, 5); (2, 4, 5); (3, 4, 5).

The NMSE was calculated in dB as shown in the equation bellow:

$$NMSE_{dB} = 10log\,\frac{\sum_{n=1}^{N}\lvert Y_{measured} - Y_{estimated}\rvert^{2}}{\sum_{n=1}^{N}\lvert Y_{measured}\rvert^{2}} \qquad (2)$$

where $Y_{measured}$ is the measured output validation value and $Y_{estimated}$ is the estimated output signal. N is the number of samples of validation, 2001.

III. RESULTS

This section presents the results of simulations conducted in a case study aimed at exploring potential enhancements in the accuracy of mathematical modeling. Across all scenarios, we considered four possibilities of models: ones where all memory values remained fixed while power values varied, ones where power values remained fixed while memory values varied, models where both power and memory varied, and models where both power and memory were fixed. Having fixed values means there are no sparsities in memory or power, whereas having varying values means there are sparsities, either in memory or in power. Since the general scenario involves variations in both values, it is expected that the most optimal results will be observed in these cases.

The utilized power amplifier is a class AB power amplifier employing a HEMT fabricated in GaN technology. Input-output data were collected using a Rohde & Schwarz FSQ vector signal analyzer (VSA) operating at a sampling frequency of 61.44 MHz. The extraction sample size was 3221, and the validation sample size was 2001 for both input and output in all cases listed above.

To assess the presumed accuracy, MATLAB with floating-point double precision was employed, alongside the least squares method using the '\' command in MALAB. The implementation of the MP, alongside with the inverse and direct modeling and the moving averages will be implemented in MATLAB and the model accuracy will be evaluated using the Normalized Mean Square Error (NMSE).

Simulation results are reported in two subsections. Subsection IV.A uses direct modeling and IV.B uses inverse modeling, both with the MP with all powers of amplitude, according to (1).

A. Direct modeling using MP with all powers

When direct modeling was used in conjunction with all powers and moving averages for the sparsed delays, the best values of NMSE that were found were for D = 2 and those values of NMSE, in dB, are reported in Table III.

Table. III. Values of NMSE, in dB, with direct modeling with moving averages.

Values of P and M	P and M are varying	P is fixed and M is varying	P is varying and M is fixed	P and M are fixed
P = 2 M = 5	-27.0296	-26.1002	-27.0296	-26.1002
P = 3 M = 3	-27.5138	-27.4464	-27.2354	-27.1287
P = 4 M = 2	-26.3039	-26.2935	-26.3005	-26.2935

After analyzing Table III, we can observe that the lowest NMSE, in dB, value is found when P is 3 and M is 3 when both power and memory values are varying.

Table IV. Values of NMSE, in dB, with direct modeling without moving averages.

Values of P and M	P and M are varying	P is fixed and M is varying	P is varying and M is fixed	P and M are fixed
P = 2 M = 5	-26.7330	-26.4295	-26.7330	-26.4295
P = 3 M = 3	-27.1624	-27.1509	-27.0535	-27.0223
P = 4 M = 2	-27.0886	-27.0513	-27.0099	-26.9820

Also, after comparing those values with the values of Table IV, where we can see the values of NMSE in dB for when moving averages weren't used [4], we can observe that when we applied moving averages, we were able to reduce the NMSE in 50% of the cases. The best case scenario was the same for both with or without the use of moving averages, when P = 3 and M = 3, but when it was used we had in NMSE decreased in 0.3514 dB.

In Figure 3, we have the output amplitude versus the input amplitude (AM-AM) plots for the best case shown in Table III, for both measured and estimated data.

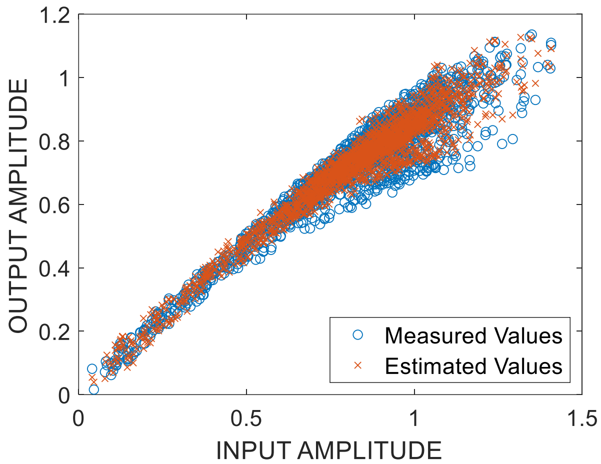

Fig. 3. AM-AM plot for the best case when direct modeling was applied with moving averages.

B. Inverse modeling using MP with all powers

When inverse modeling and moving averages for the sparsed delays were used the best NMSE was for D = 1 and those values of NMSE, in dB, are reported in Table V.

Table V. Values of NMSE, in dB, with inverse modeling with moving averages.

Values of P and M	P and M are varying	P is fixed and M is varying	P is varying and M is fixed	P and M are fixed
P = 2 M = 5	-24.5893	-24.3585	-24.5893	-24.3585
P = 3 M = 3	-24.6736	-24.4991	-23.6985	-24.6135
P = 4 M = 2	-23.9191	-23.9167	-23.7195	-23.6625

We can observe that the best value of NMSE, in dB, happened when P is 3 and M is 3, when both power and memory values were varying.

Table VI. Values of NMSE , in dB, with inverse modeling without moving averages.

Values of P and M	P and M are varying	P is fixed and M is varying	P is varying and M is fixed	P and M are fixed
P = 2 M = 5	-25.1013	-25.0277	-25.1013	-25.0277
P = 3 M = 3	-25.5866	-25.4158	-24.8342	-24.7264
P = 4 M = 2	-24.9380	-24.9380	-24.0994	-24.0994

But this time, after the use of inverse modeling with moving averages for the sparsed delays we can notice that the value of the NMSE increased in all models when compared to the values of inverse modeling for the same case but without the use of moving averages [4]. The best case scenario was the same for both with or without the use of moving averages, when P = 3 and M = 3, but when it was used we had in NMSE increased in 0.9130 dB, as it is seen on Table VI.

In Figure 4, we have the AM-AM plots for the best case when inverse modeling was used for all powers, for both measured and estimated data.

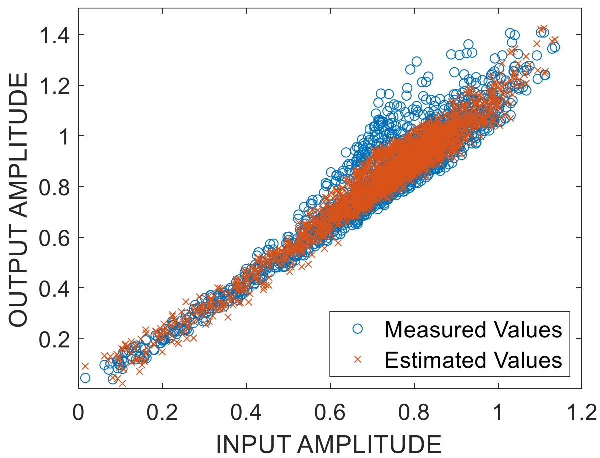

Fig. 4. AM-AM plot for the best case when inverse modeling was applied.

IV. Conclusion

The main objective of this work was to perform the mathematical modeling of a PA using MP-based models and analyze the results after using the least squares method in MATLAB, for the direct and inverse modeling, as well as using moving averages for calculating the memories values. The work presents metrics that determine which was the best modeling among the two used, direct and inverse.

The results presented show that the best NMSE was achieved when using direct modeling, specifically when the values of P and M are varying, and when the power values are equal to 3 and the memory values are equal to 3. Using moving averages helped us achieve a decreased NMSE in 0.3514 dB when compared to the same scenario, for the same modeling with the same values but without the implementation of moving averages. It is also noticeable that when inverse modeling was used, the values increased by 0.5120 dB up to 1.0213 dB compared to the values when inverse modeling was used but without the implementation of moving averages.

Acknowledgment

The authors would like to acknowledge the financial support provided by National Council for Scientific and Technological Development (CNPq) under the Program PIBITI UFPR 2022.

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